OHM'S LAW IN METALLIC CONDUCTORS

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THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

CHEMICAL ENGINEERING

COLLEGE OF LIBERAL ARTS AND SCIENCES

UNIVERSITY OF ILLINOIS

1918

UNIVERSITY OF ILLINOIS

May 9, 1918

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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ENTITLED Ohm's Law in Metallic Conductors

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Chemical Engineering

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I. INTRODUCTORY

It is a rather unusual fact that one of the oldest, best known and most widely applied of the natural laws, though empirically stated some ninety years ago, is still lacking in an established theoretical explanation. Ohm's law is an expression of the relation between the potential, quantity and resistance factors in the phenomenon of metallic conduction as determined experimentally. The actual mechanism of the process of conduction through metals together with the reasons underlying Ohm's law are yet to be completely and indisputably explained.

Inasmuch as conflicting views are still held concerning the phenomenon of which Ohm's law is an empirical statement and since certain of the proposed explanations of metallic conduction would cause us to predict departures from this law at high potential gradients it becomes a matter of interest to know whether such deviations actually occur.

It has been in the hope of taking another step toward a knowledge of the validity of Ohm's law under high potential gradients that this work has been done and while no positive results have as yet been attained at the current densities which it was hoped to reach, it is hoped that other experimenters may find some assistance in this preliminary work.

II. APPARATUS.

The problem is to measure accurately the resistance or the ratio E/I, of a metallic conductor over a wide range of current density. The difficulties involved are (1) the cooling of the conductor to remove as rapidly as possible the heat generated by the resistance of

the metal and so prevent the wire from burning out, and (2) the simultaneous measurement of the resistance and the temperature of the conductor. This is necessary in order to know whether such changes in resistance as occur are because of the temperature effect or because the ratio E/I is not a constant as stated by Ohm's law.

In undertaking this problem the work was taken up at the point where Mr. Yee working under Dr. Tolman in this (the Physical Chemistry) division, had dropped it. Possibly it should be stated here that in Mr. Yee's work and the present work the conductor used was in all cases fine copper wire since copper is easily obtainable in fine wire and easily handled and since the greater the ratio of the surface area to the cross sectional area the more rapidly the heat generated per unit volume can be removed.

Forms of apparatus previously used in experiments along this line did not prove, on trial, adaptable to any method for measuring the temperature and so the major part of the work was devoted to devising a suitable means for measuring the temperature without interfering with the measurement of the resistance.

The apparatus finally used is shown in the accompanying photograph while the essential features are shown in the diagram. The measurement of resistance is made by direct measurement of E and I by means of suitable ammeters and voltmeters. The temperature measurement depends on the thermal expansion of the wire being tested and the use of accurate means for the measurement of variations in its length.

The apparatus is constructed as follows. The fine wire whose resistance is to be measured is soldered between the heavy wire A and the wire rack (made of #18 copper wire) B which carries the stage

micrometer S. This rack is suspended by a coil spring under sufficient tension to hold the fine wire stretched tight. As the heavy wire is stationary any change in length of the fine wire must cause a vertical movement of the stage micrometer which can be measured by observation with a microscope with a cross-hair eyepiece.

The electric circuit through the fine wire is made through the heavy wire A and through two contact points pp forming an integral part of the rack and dipping into an annular mercury well to which the other connection is made.

Through the 8 mm. glass tube surrounding the fine wire transformer oil may be forced to cool the wire. The oil upon leaving the inner tube drops back into the large outer tube from which it drains back to the reservoir. A rotary motor driven pump serves to circulate the oil. The motor and pump are mounted on a stand separate from the rest of the apparatus in order to avoid vibration when making readings with the microscope.

The wire used for the experiments was #40 hard drawn copper having an average diameter of .0038 cm. An attempt to use finer wire was not successful owing to a vertical vibration of the stage micrometer which set in when a temperature of about 250° was reached. This vibration which made readings impossible in the case of the finer wire was noticeable to some extent with the heavier wire but not to a serious degree until close to the point where burning out occurred. Attempts to eliminate this vibration by using compressed air to circulate the oil (in the belief that the pump was causing the vibration) were unsuccessful and no other possible source of vibration could be discovered.

With the microscope and stage micrometer used it was found pos-

sible to measure easily changes of .01 mm. in the length of the wire. From the length of the wire (12.5 cm. in all tests) and the coefficient of thermal expansion for copper, .000018 it follows that a change of .01 mm. in length of the fine wire corresponds to a change of 4.5 degrees in temperature.

III. PROCEDURE.

Since the resistance of the fine wire alone was to be determined the first step necessary was to determine the resistance of the circuit exclusive of the test wire. This was readily accomplished by sliding the heavy wire A up through the oil tube and soldering it directly to the rack B. It will be seen that this gives identically the same circuit, minus the test wire, as the one shown in the diagram which was used in the tests themselves. Having done this a series of readings was made over the entire range of current densities, and with the same instruments later employed in the tests themselves. These measurements (see Table I) showed that the resistance of the circuit was independent of the current used, i.e. the wire used in the external circuit was heavy enough to carry the maximum current without appreciable heating. This data also furnished, of course, the resistance to be subtracted from the total measured resistance in the tests to obtain the resistance of the fine wire. The value of this constant was found to be .135 ohms.

Having completed the measurement of the resistance of the external circuit the test wire was soldered in place, care being taken to use just the length of test wire for which the temperature calibration had been calculated. The coil spring was then adjusted to the proper tension, the oil circulation started and regulated and the

microscope focused on the scale of the stage micrometer.

In making the actual test readings the current, voltage and elongation (denoted in the tables by k and read in .01 mm.) were read as nearly simultaneously as possible. The first reading was taken in every case at .3 amperes at which point there was no appreciable heating of the test wire. This reading as explained later was used as a basis for calculating the resistance of the wire at 0° C. The current was then increased at regular intervals and similar readings taken for each current until the wire burned out. Owing to the large fluctuations caused by small variations in the cooling, etc., it was never possible to get temperature readings up to the actual point where the wire gave way. The data given in Tables II, III and IV are the most reliable obtained from a series of runs.

IV. CALCULATION OF RESULTS.

From the measurements made of the current and voltage the resistance, R, was calculated directly from the relation $E/I = R$.

The temperature was calculated by multiplying the elongation k, by 4.5° and adding the product to the recorded temperature of the oil.

From the temperature of the wire as thus measured the next step was to calculate the resistance, R', which we should expect the wire to have at that temperature normally. To arrive at this value the formula expressing the resistance at a given temperature in terms of the resistance at 20° C was used. The coefficients alpha and beta for hard drawn copper were taken from the most reliable sources available but the variations in these values as determined by different investigators leaves much to be desired. From this formula,

$$R_t = R_{20}(1 + .0041(t-20) + .000005(t-20)^2)$$

the calculations were made.

As stated above the first measurement in each run showed no heating effect and so the temperature of the test wire could be assumed to be the same as that of the oil which was measured by a thermometer in the apparatus. Knowing the resistance at one measured temperature R_{20} could be readily calculated from the formula and then by use of the same formula the resistance for any temperature could be calculated. In this way the values in the column R' were calculated. It is a comparison of these values with the corresponding measured values in column R, which offers a means of arriving at a conclusion with regard to the constancy of the ratio E/I over the range of current densities employed.

V. DISCUSSION OF RESULTS.

For the range of current densities obtained in these experiments the results obtained would indicate that no appreciable deviation from Ohm's law occurs since any discrepancies in the corresponding values of R and R' fall within the range of experimental error in the experiments as conducted. The highest current density at which measurements were possible with any accuracy was in the neighborhood of 700,000 amperes per square cm.

The difficulty of choosing reliable values for the temperature resistance coefficients has been mentioned. The same difficulties were encountered in regard to the coefficient of linear expansion. These two factors alone are sufficient to account for the discrepancies in the observed and calculated values which at first glance may seem unduly great. Comparison of the results in three separate tests

shows a consistency which is good evidence of the feasibility of the method and the possibility of developing it to give more accurate results.

During the course of the experiments some question arose as to the possibility of marked differences of temperature between the center of the wire and the surface. The following considerations show that this effect is negligible.

If we consider a wire of radius <u>a</u> and length <u>l</u> whose thermal conductivity K is known, then if a current I is passed through the current density <u>i</u> will be $I/\pi a^2$. The specific resistance of the wire will be $R\pi a^2/l$ where R is the total resistance. The heat q produced per unit volume per sec. will be i^2r. If now we suppose a cylindrical boundary enclosing a central section of radius <u>a</u> and consider that a condition of equilibrium has been established at which all the heat generated in this volume is transmitted across the boundary then we have that the heat crossing the boundary per sec.

$$K\frac{dT}{da}\,2a\pi l = \pi a^2 l i^2 r$$

the heat generated per sec. since the heat transmitted across a surface per sec. equals the thermal conductivity x thermal gradient x the area.

By integrating the above equation, evaluating the constant of integration and solving for the temperature at the center T_c in terms of the temperature at the surface T_o we find that

$$T_c = T_o + \frac{1}{4K}\;\frac{I^2R}{\pi l}$$

Upon substituting known values for the case in question in this equation it was found that the maximum difference in temperature between the center of the wire and the surface was less than two degrees. It is therefore safe to assume uniform temperature conditions

throughout the wire in experiments of this type.

To increase the accuracy of the method and also to increase the workable range of current density, several improvements have suggested themselves which it has been impossible to carry out in the time available. Brief mention will be made of some of them.

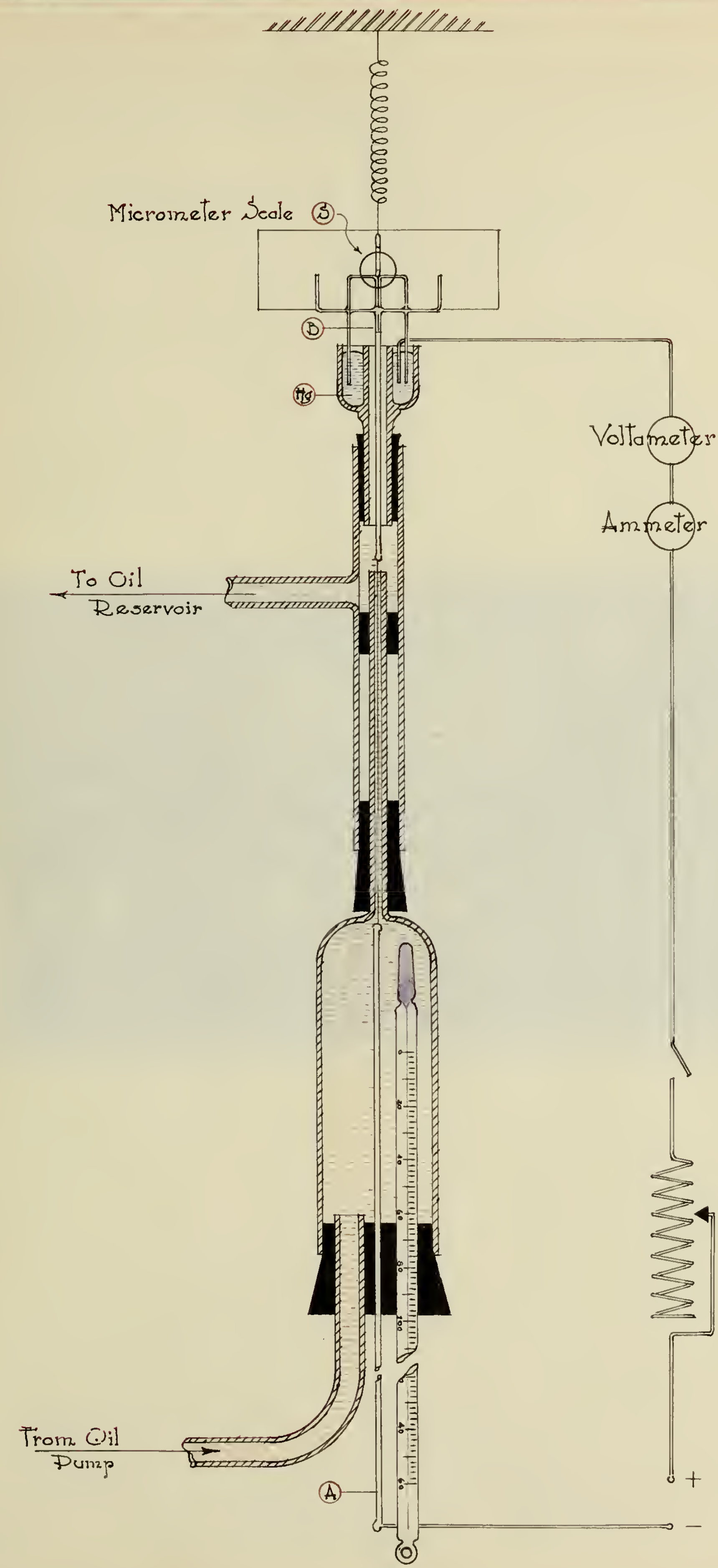
It is probable that the measurements of current and voltage can be kept up to the same degree of accuracy obtainable in the temperature measurements simply by the use of accurately calibrated and sensitive ammeters and voltmeters and by the use of a steady source of current e.g. storage batteries, with rheostats of large current capacity to avoid heating.

The improvement of the accuracy of the temperature measurements seems to depend primarily on a means of direct calibration at intervals over the range of temperatures to be measured. This has been tried but has proven impossible in the apparatus as originally constructed since the use of steam or hot oil, for example, to heat the wire results in a heating and consequent expansion of the entire apparatus which renders the results worthless. It seems entirely possible that the apparatus could be so constructed that the measurement of the elongation of the test wire would be independent of expansion of the part of the apparatus constituting the oil jacket and that expansion of the wire A could be determined and compensated. It is the writer's belief that the apparatus could be so constructed and calibrated as to give measurements of temperature accurate to within four or five degrees over a range of 500°.

Improved means of cooling are also necessary since only in that way can the range of current density be increased. Two means are possible. The first, the use of finer wire, can be accomplished if

the source of vibration can be found and eliminated. The experiments with the finer wire (.002 cm. diam.) which is probably as fine as can be manipulated, indicate that an increase of fifty percent in the current density could be accomplished by its use.

The second means is the use of a more effective cooling agent and possibly the artificial cooling of this cooling medium. The circulating system used seems to meet all the requirements since an increase in the rate of flow did not produce any noticeable change in the cooling. By the use of a more mobile liquid with a greater heat capacity and means for cooling it below room temperature, and with the finer wire it should be possible to attain current densities of two million amperes per sq. cm. under conditions in which accurate measurements could be made.



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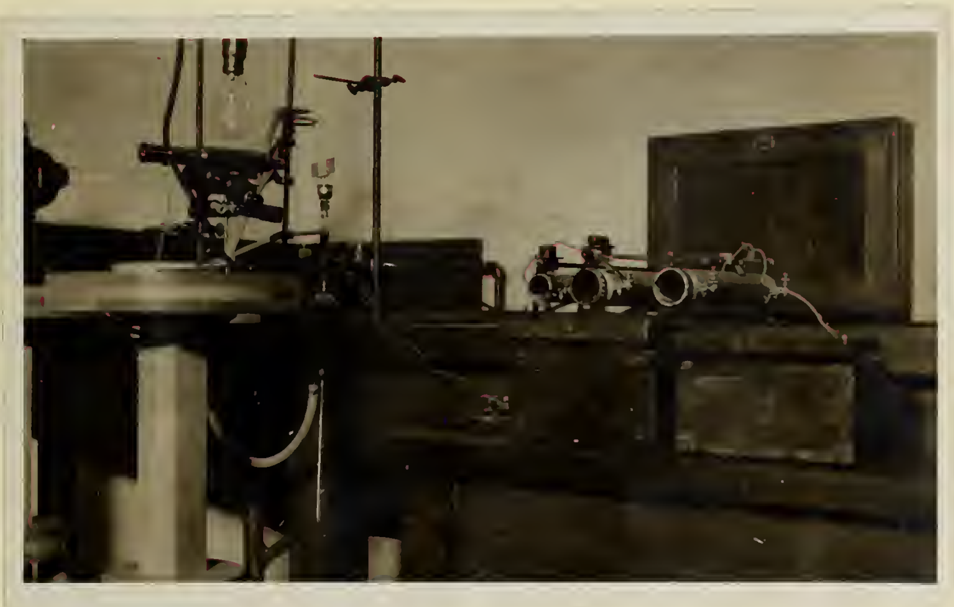

TABLE I.

Test to determine the resistance of the circuit exclusive of the test wire.

Mean value determined = .135 ohms.

E	I	R
.04	.30	.137
.07	.60	.129
.12	.90	.133
.17	1.20	.130
.20	1.50	.137
.30	2.0	.133
.40	3.0	.150
.55	4.0	.133
.65	5.0	.140
.80	6.0	.133
.90	7.0	.117
1.10	8.0	.133

TABLE II.

Test on #40 copper wire. Temperature of oil 23°.

$R = (\frac{E}{I} - .135)$

K = elongation in .01 mm.

$T = (K \times 4.5^{\circ}) + 23^{\circ}$.

R_o (calculated) = .492 ohms.

E	I	K	T	R	R'
.190	.30	.1	23	.498	.498
.250	.40	.1	23	.480.	.498
.320	.50	.6	26	.505	.505
.380	.60	.8	26	.498	.507
.450	.70	.9	27	.508	.509
.520	.80	1.0	27	.515	.509
.581	.90	1.3	29	.515	.511
.660	1.00	1.5	30	.525	.513
.730	1.10	1.8	32	.528	.517
.801	1.20	2.0	33	.531	.520
.870	1.30	2.8	36	.538	.526
.950	1.40	3.2	37	.542	.528
.103	1.50	4.0	41	.552	.535
.133	2.0	5.5	48	.530	.550
.230	3.0	15.0	95	.631	.647
.366	4.0	29.	155	.780	.769
.580	5.0	55.	273	1.025	1.099
.760	6.0	70.	338	1.135	1.280
.960	7.0	77.	368	1.235	1.375
1.130	8.0	reading of K impossible			

Wire burned out

TABLE III.

Test on #40 copper wire. Temp. of oil 23°.

$R = (\frac{E}{I} - .135)$

K = elongation in .01 mm.

$T = (R \times 4.5^{\circ}) + 23^{\circ}$.

R_{20} (calculated) = .430 ohms.

E	I	K	T	R	R'
.172	.30	.0	23	.435	.435
.231	.40	.1	23	.445	.435
.290	.50	.2	24	.445	.436
.350	.60	.3	24	.445	.436
.410	.70	.5	25	.451	.439
.470	.80	.7	26	.452	.441
.530	.90	.9	27	.453	.442
.600	1.00	1.1	28	.465	.444
.660	1.10	1.4	29	.465	.446
.720	1.20	1.5	30	.465	.447
.780	1.30	2.0	32	.465	.452
.850	1.40	2.2	33	.472	.454
.920	1.50	2.9	36	.478	.458
1.26	2.0	5.5	48	.495	.481
2.10	3.0	12.5	75	.565	.527
3.25	4.0	17.0	99	.675	.578
5.40	5.0	52.0	256	.945	.895
7.20	6.0	65.	313	1.065	1.061
8.90	7.0	75.	359	1.135	1.142
10.60	8.0	85.	405	1.185	1.300
	8.5	burned out.			

TABLE IV.

Test on #40 copper wire. Temperature of oil 23°.

$R = (\frac{E}{I} - .135)$

K = elongation in .01 mm.

$T = (R \times 4.5^{\circ}) + 23^{\circ}$.

R_o(calculated) = .443 ohms.

E	I	K	T	R	R'
.175	.30			.448	
.240	.40			.465	
.295	.50			.455	
.360	.60			.465	
.420	.70	.7	26	.465	.454
.485	.80	.9	27	.471	.456
.550	.90	1.0	27	.476	.456
.615	1.00	1.0	28	.480	.457
.680	1.10	1.5	30	.483	.462
.749	1.20	2.0	32	.489	.465
.820	1.30	2.4	34	.497	.468
.890	1.40	3.0	37	.500	.475
.955	1.50	3.5	39	.502	.477
1.30	2.0	6.0	50	.515	.495
2.22	3.0	16.5	97	.605	.591
3.60	4.0	36.0	185	.765	.778
5.80	5.0	62.0	300	1.015	1.055
7.50	6.0	77.0	368	1.115	1.238
9.60	7.0	85.0	405	1.235	1.350

Wire broke at this point